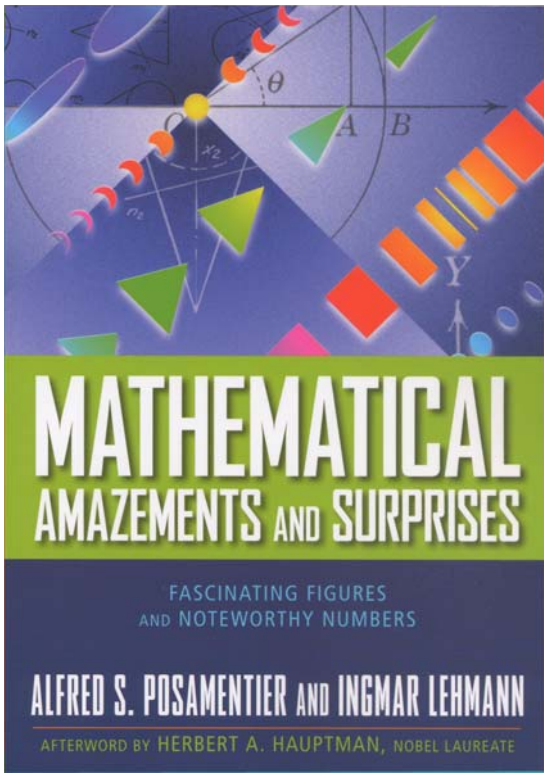

Mathematical Amazements and Surprises

Fascinating Figures and Noteworthy Numbers

Alfred S. Posamentier & Ingmar Lehmann

Afterword by Herbert Hauptman, Nobel Laureate

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Preface

Too often mathematics is looked upon as a sequential arrangement of topics that seems to lead nowhere useful. Every once in a while, we are given some applications where the mathematics learned in school comes in handy, such as calculating the shortest path between two locations or determining the best price for various quantities of the same product. Of course, for those who work in a technical setting—science, engineering, economics, just to name a few—there is a constant need to use mathematics. Most people's introduction to mathematics has been bereft of exhibiting its beauty through the many unusual relationships within and between common topics. This book is designed to open up the world of mathematical wonders through largely numerical and geometrical phenomena. Naturally, there will be times when we will consider topics just a bit beyond simple arithmetic, such as some probability surprises, or use some algebra to justify some
of these oddities. Yet throughout we will be cognizant of the general readership—keeping the language and the nomenclature simple so that it can be easily understood.

For example, we will consider one of the most counterintuitive illustrations in mathematics— in this case using probability. What do you think the likelihood is of finding two people in a room of thirty-five people who share the same birth date (not necessarily the same year)? Our intuition would dictate that it is quite unlikely, or the probability is rather low. Well, brace yourself; the actual probability is somewhat more than 80 percent! This is just one of the many surprises in store for you in this book.

There are many entertaining shortcuts in arithmetic or in recognizing number relationships at a glance, such as visually determining by what numbers a given number may be divided exactly. There are lots of unusual properties of specific numbers in our number system—many of which can be quite engaging, besides being useful. Exploring these will pique your curiosity and improve you insights into the nature of numbers. Here again, we will be gently building your appreciation for the beauty of mathematics.

When one speaks of beauty in mathematics, one might expect it to be visual. Of course, geometry lends itself quite nicely to that approach. There are many situations in which you begin with a rather general geometric shape and, by following simple consistent procedures, you end up—quite unexpectedly—with a beautiful geometric shape. For example, if you draw any (ugly) quadrilateral and join the midpoints of the sides, consecutively, with line segments, you will *always* end up with a parallelogram—sometimes a rhombus, other times, a square or a rectangle. We will show you shapes that are universally deemed beautiful, such as the golden rectangle, and show you how to easily construct them. The beauty that geometry provides for mathematics is boundless, so we present only some of the highlights for you to consider—mostly without proof that they are, in fact, true. Instead of just taking our word for their truth, we urge you to try to prove the relationships true. For this we provide you with an extensive bibliography. Beyond the beauty of the relationships lies the true beauty of the proofs that establish these amazing geometric marvels.

In short, this book will take you through a plethora of amazing and surprising examples of mathematical phenomena, each of which will contribute to your accepting our premise that there is real beauty — both visual and intellectual — in mathematics to be savored.

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Errata

Page 22: Line 8. Change 127 to 126.

Page 131: Line 6. Change 32,**9**67 to 32**9**67.

We appreciate any comments about the book as well as any typographical errors that have not yet been detected so that they can be incorporated in future printings of the book.

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