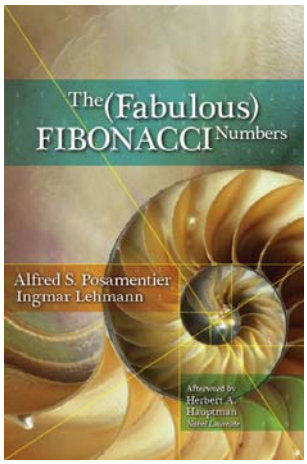

The (Fabulous) Fibonacci Numbers

Alfred S. Posamentier & Ingmar Lehmann

Afterword by Herbert Hauptman,
Nobel Laureate

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Publisher's notes (On Book Jacket)

The most ubiquitous, and perhaps most intriguing, number pattern in all of mathematics is the Fibonacci sequence. In this strikingly simple pattern, beginning with two ones, each succeeding number is the sum of the two numbers immediately preceding it (1, 1, 2, 3, 5, 8, 13, 21, ad infinitum). Far from being just a mathematical curiosity, however, this sequence recurs throughout nature—from the regeneration patterns of bees and rabbits to the arrangement of spirals on pinecones and pineapples. All of which is astounding evidence for the deep mathematical basis of the natural world.

With admirable insight and clarity, math educators Alfred Posamentier and Ingmar Lehmann take us on an utterly fascinating tour of the many ramifications of the Fibonacci numbers. The authors begin with a brief history of their distinguished thirteenth-century Italian mathematician Leonardo of Pisa (more commonly known as Fibonacci), whose other accomplishments including popularizing the use of Arabic numerals in the West. Turning to the field of botany, the authors demonstrate through illustrative diagrams, the many amazing connections between the Fibonacci numbers and natural forms (including pinecones, pineapples, sunflowers, and daisies). In art, architecture, the stock market, and others areas of society and culture, Posamentier and Lehmann find an almost endless array of instances where the Fibonacci sequence, as well as its derivative, the "golden ratio, " makes an appearance. And, of course, as the authors amply demonstrate, there are almost boundless applications in probability, algebra, and Pascal's triangle, to name but a few.

Thoroughly accessible and appealing to even the math-phobic individual, this fun-filled and enlightening book allows the reader to appreciate the true elegance of mathematics and its amazing applications in the world around us.

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$\approx 3.7\ 71494711 \cdot 10^{388}$

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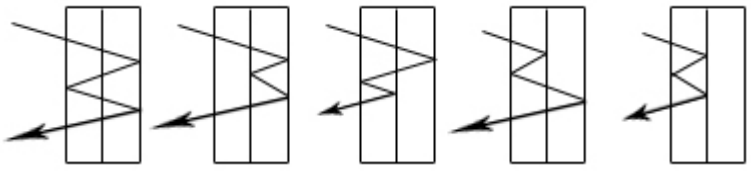

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We appreciate any comments about the book as well as any typographical errors that have not yet been detected so that they can be incorporated in future printings of the book.

Alfred S. Posamentier: asp2@juno.com
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Ingmar Lehmann: ilehmann@mathematik.hu-berlin.de
Humboldt University of Berlin