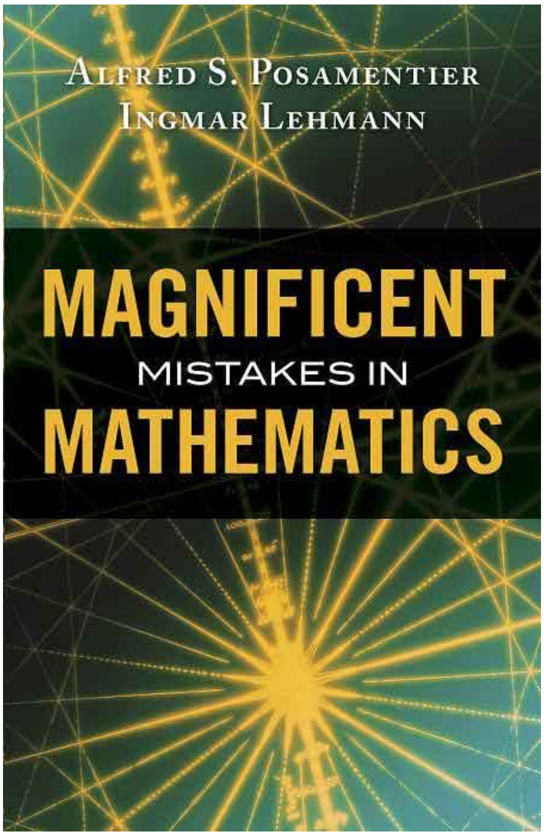
Errata and more:



Magnificent Mistakes in Mathematics

Alfred S. Posamentier & Ingmar Lehmann

Amherst (New York), Prometheus Books,

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Introduction

The title of this book can be interpreted in a number of ways. In the study of mathematics we have all made mistakes along the way. We are not referring to the kind of errors due to carelessness, or lack of understanding or even silly errors of notation.

Some mistakes are merely well-hidden subtle errors. Take for example the English mathematician William Shanks (1812-1882) who required 15 years for his calculation of the value of π to set the record for the number of decimal places in 1874. In 1937, in Hall 31 of the Palais de la Decouverte (today a Paris science museum on Franklin D. Roosevelt Avenue), this value of π was produced with large wooden numerals on the ceiling (a cupola) in the form of a spiral. This was a nice dedication to this famous number, but, surprisingly, there was an error. Shanks' approximation was discovered to have contained a mistake, which occurred at the 528th decimal place. This was first detected in 1946 with the aid of a mechanical desk calculator – using only 70 hours of running time! This error in the "π room" on the museum's ceiling was corrected soon thereafter in 1949. The race for accuracy for the value of π is today in the trillions of decimal places[1].

While we speak of mistakes on public display, consider the clock atop the tower on St. Marien Church, the oldest structure in the town of Bergen on the German island of Rügen in the Baltic Sea. Damaged during a storm in 1983, the clock was restored in 1985 by craftsmen who found themselves with a conundrum. As they were inserting the minute markers on the face of the clock they found that there was a large and unexpected space between the 11 and 12. So, they simply inserted another marker to fill the space. Consequently this may be the only clock in the world with 61 intervals on its face rather than the proper 60 intervals. (See figure I-1.)

Figure I.1.
Courtesy of Norbert Rösler, sexton of the church St. Marien, Bergen / Rügen (Germany)

There is also a clever story about a standardized test that presented two pyramids: one comprised of four equilateral triangular faces (a regular tetrahedron) and the other a pyramid with a square base and lateral sides that were equilateral triangles congruent to those of the regular tetrahedron. Students were asked to place the two pyramids together by overlapping their mutually congruent equilaterals faces, and then determine how many faces the resulting figure had. Given that the sum of the faces of the two pyramids is 4 + 5 = 9, and by overlapping two of these faces eliminates them, the "correct" answer was given to be 7 faces. It was not until some years later that a student persevered to show that in fact, this was a wrong answer, since by this combination of pyramids two rhombuses are formed by adjacent equilateral triangles, thus resulting in only five faces. We will take a closer look at this situation in chapter 4. Such errors give us pause to examine even the "obvious."

[1] See Alfred S. Posamentier; Ingmar Lehmann: *Pi: A Biography of the World's Most Mysterious Number*. Afterword by Herbert Hauptman, Nobel Laureate. Amherst (New York), Prometheus Books, 2004, p. 70ff.

Our objective in this book is to entertain the reader with a collection of wrong conclusions – or fallacies – that help us to better understand important aspects or concepts in mathematics. It is through these "mistakes" that we should get a much better insight of and appreciation for of the subject matter. Some "mistakes" can lead to very interesting mathematical ideas. For this reason, we have deemed them "magnificent mistakes." But rest assured that no special mathematical skills are needed to explore with us these fascinating mistakes. We expect the reader to know high school mathematics and nothing necessarily beyond that.

There are some simple mistakes we make that because they lead to an absurd result, and we tend to dismiss the mistake. We know that if equals are multiplied by equals, then the results are equal. For example, if we know that $x = y$, then we also can conclude that $3x = 3y$. Yet, when we have 2 pounds = 32 ounces, and $\frac{1}{2}$ pound = 8 ounces, then does $2 \cdot \frac{1}{2}$ pound = $32 \cdot 8$ ounces? Or does 1 pound = 256 ounces? Of course not. Where did we go wrong? Similarly, we know that $\frac{1}{4}$ dollar = 25 cents. Then does $\sqrt{\frac{1}{4} \text{ dollar}} = \sqrt{25 \text{ cents}}$, or $\frac{1}{2}$ dollar = 5 cents? Again, absurd! Where did we go wrong? When we multiplied the numbers or took their square root, we didn't do that to the units, which would have led us to a correct solution – albeit an awkward one! To make this explanation a bit simpler, suppose we begin with 2 feet = 24 inches, and $\frac{1}{2}$ foot = 6 inches; then, by multiplying the units and the measures, we get 1 square foot = 144 square inches, which is correct!

We can also consider how the "proof" that 1 = 0 leads us to a most important mathematical concept: that division by zero is not permissible. Follow along as we show this interesting little "proof." We begin with our given information that $x = 0$. We then multiply both sides of this equation by $x - 1$ to get $x(x - 1) = 0$. Now dividing both sides by x leaves us with $x - 1 = 0$, which in turn tells us that $x = 1$. However, we began with $x = 0$. Therefore, 1 must equal 0. Absurd! Our procedure was correct. So why did we end up with an absurd result? Yes, we divided by zero when we divided both sides of the equation by x. Division by zero is not permitted in mathematics, as it will lead us to silly conclusions. This is just one of many such entertaining mistakes that give us a more genuine understanding of the "rules' of mathematics.

These examples may seem entertaining, and they are. Yet through these entertaining illustrations of mistakes a lot is to be learned about mathematical rules and concepts. For example, when we "prove" that every triangle is isosceles, we are violating a concept not even known to Euclid – that of betweenness. When we show that the sum of the lengths of two legs of a right triangle is equal to the length of the hypotenuse – clearly violating the time-honored Pythagorean theorem – we will be showing a misuse of the concept of infinity. Yet, it is the unique value of these mistakes – providing a better understanding of the basic concepts of mathematics – that makes these mistakes magnificent. Lest we forget, youngsters – and we dare say, adults as well – learn quite a bit from mistakes. We expect that through the playful style in which we present these mistakes the reader will be delightfully informed! We shall also compare mathematical mistakes with those in everyday life and notice what can be learned from these.

We expect that the readers will enjoy these examples, and during this delightful excursion they should appreciate the many aspects or nuances of mathematics that sometimes go unnoticed until they lead one astray. We invite you now to begin your journey through these many magnificent mistakes in mathematics.

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Errata

Page 21, Line 15: Change “1676” to “1657”.

Page 23, Line 7 from bottom, to Page 24, Line 2:

Page 23, line 4+: Replace the following sentences:

> “He noticed that the time of the ball’s travel was inversely proportional to the number of vertices of the polygonal path. The more vertices the path has, the less travel time is required. Recognized that by constantly increasing the number of vertices, the polygon surface will approach the arc of a circle, he therefore conjectured that the arc of a circle must be the fastest curve for the ball to travel rather than a straight line.
>
> What he didn’t consider was that the polygon sides do not necessarily have the same length, and, therefore, do not approach the curve of the circle.”

With the following:

> “He established that as the number of linear paths along the ball‘s polygonal travel increased, the time decreased, and then correctly concluded that the closer the sum of the linear portions approaches the circular arc, the less time it will take for the ball to travel from *A* to *B*. Subsequently, Galileo believed that the circular path was the fastest path. This conclusion is false; this is where Galileo erred.”

Page 25, Line 11: Replace the following: “With this thinking, he lost lots of money and desperate as he was, …”

With the following: to “With this thinking, he thought to have lost lots of money and desperate as he was, …”.

Page 34, Line 8 from bottom: add the death date as follows: Kenneth Appel (1932-2013)

Page 36, Line 5 from bottom: Alphonse de Polignac (1817-1890) dates should be (1826-1863)

Page 39, Line 9 from bottom: Change "November 18, 1852" to "November 18, 1752".

Page 84, Line 7: Change "There are simple algebraic methods for determining the irrationality of a number." to "There are – in some examples – simple algebraic methods for determining the irrationality of a number."

Page 136, Lines 3 and 2 from bottom: Sentence should read:

The error lies in the misinterpretation of the words *have* and *has* as implying the same as *equals*.

Page 159, Figure 4.27, column (3): Change "$\varphi \approx 126.9°$, $\psi \approx 143.1°$" to "$\varphi \approx 143.1°$, $\psi \approx 126.9°$".

Page 195, First paragraph should be replaced with the following:

You are seated at a table in a dark room. On the table, there are 12 pennies, 5 of which are heads up and 7 are tails up. Now mix the coins and separate them into two piles of 5 and 7 coins, respectively. Because you are in a dark room, you will not know if the coins you are touching were heads up or tails up. Then flip over the coins in the 5-coin pile. When the lights are turned on there will be an equal number of heads in each of the two piles. How can this be possible?

Page 195, Line 12: Change "conference" to "circumference".

We appreciate any comments about the book as well as any typographical errors that have not yet been detected so that they can be incorporated in future printings of the book.

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