

STUDY GROUP ON GEOMETRY AND ARITHMETIC OF K3 SURFACES

Winter Semester 2012 – 2013, Humboldt University

Organised by: Anil Aryasomayajula, Frank Gounelas
Supervised by: Jürg Kramer

Time and place: At 11am on Tuesdays in room 1.023 (Haus 1, Raum 023 ground floor) at 25 Rudower Chaussee.

1. ORGANISATIONAL

We aim to meet once a week and have a one and a half hour talk given by a different person each week. The talks are encouraged to be relatively informal in style, encouraging discussion. Speakers should try and give proofs wherever possible, especially in the first half of the semester where the material will be more tractable.

2. CONTENTS

The tentative plan is to begin with the first six or so talks introducing enough basic theory of surfaces so as to define K3 surfaces, first over the complex numbers and then separately in positive characteristic. For this, an option is to follow [Huy10] which also contains an extensive list of references on the topic. Afterwards, the goal will be to discuss current popular trends in the geometric and arithmetic theory of K3 surfaces. The contents of the latter talks will depend on the preferences of the audience participating, so suggestions are welcome! The first direction which we will aim to cover before the holiday break is the following

Geometry of curves on K3 surfaces, following the recent results of [BHT11] and [LL10] proving that complex K3 surfaces of odd Picard rank contain infinitely many rational curves. Topics would include the deformation theory of curves in surfaces and Mori-type reduction mod p arguments. See the notes of Hassett [Has10] and Chen's papers [Che99], [Che02].

A final selection of further topics remains to be discussed with the participants, but some possibilities are the following (in no particular order of importance)

- The Weil conjectures for K3s. See Deligne's paper [Del72].
- Lifting K3 surfaces from positive characteristic to characteristic zero. See Deligne [Del81].
- Recent work in [LMS11], [Mau12], [Cha12] proving the Tate conjecture for K3 surfaces over finite fields of characteristic $p > 5$.

- Batyrev's paper [Bat99] proving that if X, Y birationally equivalent Calabi-Yau varieties, then the Betti numbers $b_i(X) = b_i(Y)$. This uses the Weil conjectures and mod p arguments. See also notes by Popa
<http://homepages.math.uic.edu/~mpopa/571/>
- Mumford on $CH^2(S)$ and Bloch-Beilinson for K3s. See also [Voi07a], [Voi07b].
- Rational points on K3 surfaces over number fields. Density results. See [Sch08].
- Derived equivalences and K3 surfaces. See work of Bondal/Mukai/Orlov e.g. in Huybrecht's book [Huy06].
- Moduli of K3s [GHS11] and Voisin's Bourbaki talk
<http://www.math.jussieu.fr/~voisin/Articlesweb/Exp.981.C.Voisin.pdf>.

3. CURRENT PLAN OF TALKS

3.1. 23 October : Introductory talk. Speaker: Frank Gounelas

Outline and organisational.

3.2. 30 October : Generalities on surfaces I. Speaker: Ana Maria Botero [Har77], [Mum66], [Huy05], [Fri98], [BHPVdV04]. Line bundles, divisors, linear systems, intersection form, adjunction, Riemann-Roch, Nakai-Moishezon and Kleiman. It's recommended to have a look at the following notes [Hul02], [Pet04b], [Pet04a], [Deb10].

3.3. 6 November : Generalities on surfaces II. Speaker: Stefan Keil [Har77], [Bea96], [Rei93], [BHPVdV04], [Mum66], [Fri98]. Hodge numbers/diamond (Serre and Hodge duality) and other invariants: b_i, p_g, q , Kodaira dimension, the classification. It's recommended to have a look at the following notes [Hul02], [Pet04b], [Pet04a], [Deb10].

3.4. 13 November : Complex K3 surfaces I. Speaker: Ronnie Sebastian [BHPVdV04], [Huy10], [Mor88]. Definitions, computation of invariants, projective embeddings, polarisations, Saint-Donat's Theorem. Almost all this is in chapters 1, 2 of Huybrecht's notes.

3.5. 20 November : Characteristic p K3's. Speaker: David Ouwehand [Huy10], [Art74], [Has10]. Artin's conjectures, even Picard number over $\mathbb{F}_p$ with $p \geq 5$, supersingular K3s, reduction and lifting and the curious case of the Fermat quartic in $\mathbb{P}_{\mathbb{F}_3}^3$ (which is unirational!).

3.6. 27 November : Complex K3's II. Speaker: Giovanni De Gaetano [BHPVdV04], [Huy10], [Mor88], [Voi07a], [Voi07b]. Lattices, Hodge structures and statement of Torelli. Basics of deformation theory (Kodaira-Spencer) and the moduli of polarised K3s.

3.7. 4 December : Rational curves on K3s: Deformations of stable curves. Speaker: Fabian Müller [Kol96], [Deb01], [BHT11]. Deformations of curves, stable morphisms: section 4, Proposition 15, Theorem 18 of [BHT11] (nb numbering from Duke paper and not arxiv version).

3.8. 11 December : Rational curves on K3s: Mori-Mukai on constructing rational curves. Speaker: Barbara Jung

[BHT11], [Has10], [MM83]. The Mori-Mukai argument given in [Has10] and the arxiv version of [BHT11].

3.9. 18 December : Rational curves on K3s: Bogomolov-Hassett-Tschinkel on constructing rational curves. Speaker: Niels Lindner

Producing rational curves in positive characteristic, Proposition 17 and proof of main theorem from [BHT11].

3.10. 8 January : Weil conjectures for curves I. Speaker: Ananyo Dan

Following Sam Raskin's notes [Ras07], introduction to Weil conjectures, zeta functions, rationality, eigenvalues of Frobenius...

3.11. 15 January : Weil conjectures for curves II. Speaker: Inder Kaur

Following Sam Raskin's notes [Ras07], the proof in the case of curves.

3.12. 22 January : Etale and l -adic cohomology. Speaker: Giulia Battiston

Why we need it, Poincaré duality, Lefschetz Trace formula and other desirables for the conjectures.

3.13. 29 January : Weil conjectures (assuming RH). Speaker: Angela Ortega

Statements and proofs of the general case following Milne [Mil12] chapter 27.

3.14. 5 February : Modularity and K3s I. Speaker: Anil Aryasomayajula

Some new results of Noam Elkies and Matthias Schütt from [ES] on modularity of K3 surfaces

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