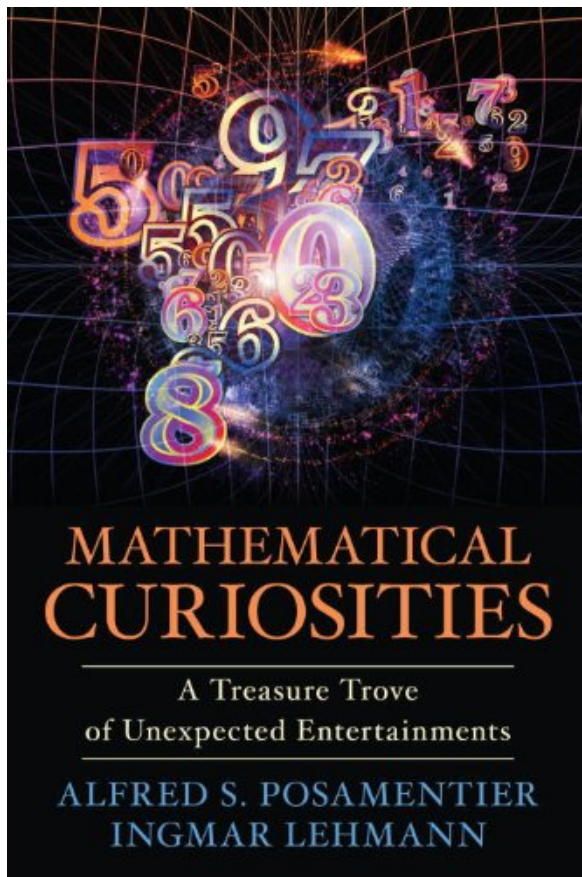


Errata and more:



## Mathematical Curiosities

A Treasure Trove  
of Unexpected Entertainments

Alfred S. Posamentier &  
Ingmar Lehmann

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## Ciekawostki matematyczne

Skarbnica zadziwiających rozrywek

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## Introduction

It is unfortunate that too many people would be hard-pressed to consider anything mathematical as entertainment. Yet with this book we hope to convert the uninitiated general readership to an appreciation for mathematics – and from a very unusual point of view: through a wide variety of mathematical curiosities. These include, but certainly not limited to, peculiarities involving numbers and number relationships, surprising logical thinking, unusual geometric characteristics, seemingly difficult (yet easily understood) problems that can be solved surprisingly simply, curious relationships between algebra and geometry, and an uncommon view of common fractions.

In order to allow the reader to genuinely appreciate the power and beauty of mathematics, we navigate these unexpected curiosities in a brief and simple fashion. As we navigate through these truly amazing representations of mathematics, we encounter in the first chapter patterns and relationships among numbers that a reader on first seeing these will think they are contrived, but they are not. It is simply that we have dug out these morsels of fantastic relationships that have bypassed most of us during our school days. It is unfortunate that teachers don't take the time to search for some of these beauties, since students in their development stages would see mathematics from a far more favorable point of view.

During the centuries of isolation, the Japanese population was fascinated with *Sangaku* puzzles, which we will admire in chapter 2 for the geometry they exhibit. It will allow us to see a curious side of geometry that may have passed us by as we studied geometry in school. We use these puzzles as a gateway to look at some other geometric manifestations.

Problem solving, as most people may recall from their school days, was presented in the form of either drill questions or carefully categorized topical problems. In the case of drill, rote memorization was expected, while in the case of topical problems, a mechanical response was too often encouraged by teachers. What was missing were the many mathematical challenges – problems in a genuine sense – that are off the beaten path, that do not necessarily fit a certain category, that can be very easily stated, and that provide the opportunity for some quite surprisingly simple solutions. These problems, provided in the third chapter, are intended to fascinate and captivate the uninitiated!

Measures of central tendency have largely been relegated to the study of statistics – as well they might be. However, when seen from a strictly mathematical point of view – algebraic and geometric – they provide a wonderful opportunity for geometrically justifying algebraic results or algebraically justifying geometric results. We do this in chapter 4, largely in the context of comparing the relative sizes of the four most popular means, or measures of central tendency, namely, the arithmetic mean (the common average), the geometric mean, the harmonic mean, and the root-mean-square.

When fractions are taught in school, they are presented largely in the context of doing the four basic arithmetic operations with them. In our last chapter, we present fractions from a completely different standpoint. First recognizing that the ancient Egyptians used only unit fractions (i.e., those with the numerator is 1) and the fraction  $\frac{2}{3}$ . We will present unit fractions in a most unusual way as part of a harmonic triangle and eventually leading to *Farey sequences*. The reader should be fascinated to witness that fractions can be more than just representing a quantity and being manipulated with others.

Every attempt has been made to make these curiosities as reader friendly, attractive and motivating as possible so as to convince the general readership that mathematics is all around us and can be fun. One by-product of this book is to make the reader more quantitatively and logically aware of the world around him or her.

There are numerous examples in this treasure trove that we hope to have presented in a highly intelligible fashion so as we to rekindle a readers' true love for mathematics, and that for those who might have been a bit skeptical about curiosities that can exhibit the power and beauty of mathematics will now go forward and serve as ambassadors for the great field of mathematics. One of our goals in this book is to convince the general populace that they should enjoy mathematics and not boast of having been weak in the subject during their school years.

# Mathematical Curiosities

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By Alfred S. Posamentier and Ingmar Lehmann  
Prometheus Books, 2014

### Errata

Page 19, Line 3 from the bottom: change  $\frac{6846}{8648}$  to  $\frac{6486}{8648}$ .

Page 39, Line 5/6: change “in  $\frac{72}{r} = n$  years” to “in  $\frac{72}{8} = 9$  years”.

Page 42, Line 1-8: replace all the Hebrew characters for ז (Zayin) with the Hebrew character ו (Vav). That is the second letter from the right – in ז ו ז and in ו ז.

Page 46, the next-to-last line should not read (24, 25). It should read (8, 9).

Page 97, Line 11/12: replace with the following:  
“number  $4^2 = 16$ , and take  $2 \cdot 4 + 1 = 9$ , and add this to the square, we get the next square number,  $16 + 9 = 25 = 5^2$ ”.

Page 297, Line 3 from the bottom: replace the inequality with:  $\frac{2ab}{a+b} \leq \sqrt{a \cdot b} \leq \frac{a+b}{2}$ .

Page 301, Line 4 from the bottom: change “ $\frac{d}{2} = \sqrt{2} \cdot \sqrt{a^2 + b^2}$ ” to “ $\frac{d}{2} = \frac{\sqrt{2}}{2} \cdot \sqrt{a^2 + b^2}$ ”.

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We appreciate any comments about the book as well as any typographical errors that have not yet been detected so that they can be incorporated in future printings of the book.

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