Seminar on arithmetic geometry: Deligne-Mumford stacks

Prof. Dr. Jürg Kramer
Marco Flores
Gari Peralta

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Introduction

An *algebraic stack* is a generalisation of the concept of *scheme*. While the theory of schemes proved to be useful to describe a wide class of phenomena in algebraic geometry, it lacked the tools to handle a central problem in mathematics: *The moduli problem.* In their celebrated article [DM69], Deligne and Mumford introduced the concept of a *"Deligne-Mumford stack"* to describe the isomorphism classes of curves of a given genus as a geometric space satisfying nice properties. This was later generalised by Artin [Art74].

Description

The goal of this seminar is to give an introduction to the theory of stacks through the moduli problem. The theory of stacks is rather infamous for being too technical. To balance this difficulty, we will focus on working out examples: the moduli spaces $\mathcal{M}_g$ and $\mathcal{A}_g$ of curves of genus g and (principally polarised) abelian varieties of dimension g respectively.

- **Prerequisites**: A solid background in commutative algebra and algebraic geometry, for example, proficiency with chapters 1 to 3 of Hartshorne's *Algebraic geometry.*
- **Structure**: The seminar will be divided into 10 talks. A preliminary division can be found below. To approve this course, the participant must give (at least) one talk. Students that do not wish to obtain credits for this course are still welcome to attend, without the obligation of giving a talk.
- **Inquiries**: For questions or suggestions, please write a short email to peraltag@hu-berlin.de.

Talks

1. **Introduction and motivation** (Marco Flores - 02.11.22).
2. **Sites, sheaves and stacks** (Miguel Carbajo - 09.11.22): Summary of chapter 1 of [Alp21].
3. **Algebraic stacks** (Gari Peralta - 16.11.22): Section 2.1 of [Alp21]. Summary of B3 of [Alp21].
4. **First properties** (Shi Yu - 23.11.22) : Sections 2.2 to 2.4 of [Alp21].
5. **Dimension and tangent spaces** (Branislav Sobot - 30.11.22): Section 2.5 of [Alp21]. Explain the case of $\mathcal{A}_g$ as in Ch. I, 4.3 and 4.11 of [FCh90]. See also 2.1 of the survey [OlsAb].

6. **D-M stacks, smoothness and properness** (Zongpu Zhang - 07.12): Sections 2.6 to 2.8 of [Alp21].
7. **The moduli stack of stable curves** (Li Li - 11.01.23): Chapter 4 of [Alp21].
8. **The moduli stack of formal group laws** (Mark Backhaus - 18.01.23).
9. **The geometry of D-M stacks** (Paul Brommer-Wierig - 01.02.23).

References

[Alp21] J. Alper; *Course lecture notes*, `https://sites.math.washington.edu/~jarod/math582C-winter21/moduli-6-24-21.pdf`.

[Art74] M. Artin; *Versal deformations and algebraic stacks*, Invent. Math., 27: 165–189, 1974.

[DM69] P. Deligne, D. Mumford; *The irreducibility of the space of curves of given genus*, Publications mathématiques de l'I.H.É.S., tome 36 (1969), p. 75-109.

[Ed03] D. Edidin; *What is... a stack?*, Notices of the AMS, volume 50, number 4, 2003. `https://www.ams.org/notices/200304/what-is.pdf`

[FCh90] G. Faltings, C.-L. Chai; *Degeneration of abelian varieties*, Ergebnisse der Mathematik und ihrer Grenzgebiete. 3. Folge, Springer-Verlag Berlin Heidelberg 1990.

[Ols16] M. Olsson; *Algebraic spaces and stacks*, American Mathematical Society, 2016.

[OlsAb] M. Olsson; *Compactifications of moduli of abelian varieties: An introduction*, `https://math.berkeley.edu/~molsson/Overview3.pdf`.